Efficient Use of an Intermediate Reboiler or Condenser in a Binary Distillation

Rakesh Agrawal and D. Michael Herron
Air Products and Chemicals, Inc., Allentown, PA 18195

The impact of an intermediate reboiler or condenser on the distillation of ideal binary mixtures into pure product streams is studied using a simplified model. The advantage of heuristics derived from this study is that they can quickly tell a process engineer if an intermediate reboiler or condenser is going to be effective in improving the efficiency and, of the two options, which one would be more effective. The heuristics simply states that if the actual fraction of liquid in a given feed is less than that with the maximum thermodynamic efficiency for distillation with no intermediate reboiler or condenser, then an intermediate condenser not only substantially improves the thermodynamic efficiency but is also more effective than an intermediate reboiler. An analogous heuristics exists for the intermediate reboiler when the fraction of liquid in the feed is greater than the optimum. Quick identification of cases that can achieve a substantial improvement in efficiency provides an incentive to search for the proper utilities needed for the intermediate reboiler or condenser. When relatively pure feed streams (concentration of either component greater than 90%) are distilled, the extremely low efficiencies of distillation can be remarkably improved by using an intermediate reboiler or condenser.

Introduction

It is well known that the thermodynamic reversibility of a binary distillation column can be increased by using intermediate reboilers and intermediate condensers (King, 1980). In a conventional column with one feed, a distillate, and a bottoms product, heat is supplied at the highest temperature to a reboiler at the bottom of the column and is removed at the lowest temperature from a condenser at the top of the column. This requires that all the heat utility be available at a temperature warmer than the bottom reboiler temperature and the cold utility be available at a temperature cooler than the top condenser temperature. With intermediate reboilers, some of the heat of the distillation column can be provided at temperatures lower than the bottom reboiler temperature. Similarly with intermediate condensers, some of the cooling duty can be provided at higher temperatures than the top condenser. Such near-reversible binary distillation columns have been analyzed in the past (Benedict, 1947; Flower and Jackson, 1964; Fonyo, 1974; Fitzmorris and Mah, 1980; Koehler et al., 1991). A totally reversible distillation column would require infinite numbers of heat exchangers and separation stages.

Although reversible distillation is not commercially attainable, in practical applications the irreversibilities can often be significantly reduced by applying just one intermediate reboiler or one intermediate condenser (Terranova and Westerberg, 1989; Fidkowski and Agrawal, 1995). When a fraction of the total heat requirement is supplied by a heat source with a temperature lower than that of the bottom reboiler, it may be feasible to use an inexpensive "waste energy" stream as this heat source (Patterson and Wells, 1977). When a heat pump, such as a vapor recompression and condensation cycle, is used with an intermediate reboiler, then the total compression energy is decreased (Flower and Jackson, 1964; Lynd and Grethlein, 1986). Similarly, the use of an intermediate condenser can enable a portion of the condensation duty to be provided by a less expensive cold utility, or the total energy consumption can be reduced if heat pumps are used. For gas separations by distillation at below ambient temperatures, intermediate reboilers and intermediate condensers are often used to reduce compression energy consumption (Woodward and Agrawal, 1990).

Correspondence concerning this article should be addressed to R. Agrawal.

In spite of the potential of an intermediate reboiler or an intermediate condenser to improve the performance of a binary distillation column, only a few guidelines are currently available to help a process designer. This is reflected by two recent publications (Wankat and Kessler, 1993; Fidkowski and Agrawal, 1995). The objective of this work is to provide some of the needed heuristics on the use of an intermediate reboiler or an intermediate condenser to distill an ideal binary mixture into pure products. For a given feed, several pertinent questions are answered: When does the use of an intermediate reboiler or an intermediate condenser provide meaningful improvement in thermodynamic efficiency? When is an intermediate reboiler more effective than an intermediate condenser and vice versa? Where is the most efficient location for an intermediate reboiler or an intermediate condenser? Quick answers to these questions can help a process engineer to synthesize efficient distillation schemes. While the first two questions are answered in this article, the last question on the optimum location for an intermediate reboiler or an intermediate condenser is answered in the accompanying publication (Agrawal and Herron, 1998).

The maximum efficiency gain possible by using an intermediate reboiler or an intermediate condenser is calculated for several feed conditions. This is done for pinched columns distilling ideal binary mixtures with constant relative volatilities, that is, mixtures exhibiting inflection points on their $y-x$ equilibrium diagrams are excluded. A close examination of these results provides answers to all the questions raised.

Maximum Efficiency with an Intermediate Reboiler or an Intermediate Condenser

The use of an intermediate reboiler in the stripping section and an intermediate condenser in the rectifying section of a binary distillation column is illustrated in Figure 1. It is well known that when an intermediate reboiler is used, the total heat requirement does not change; rather a portion can be applied at a lower temperature in the intermediate reboiler (Terranova and Westerberg, 1989; Fidkowski and Agrawal, 1995). This leads to an increase in thermodynamic efficiency. As the location of the intermediate reboiler is moved up closer to the feed, however, the quantity of heat input to the intermediate reboiler (Q_{IR}) decreases and greater proportion of heat at the higher temperature is required in the bottom reboiler (Q_B). As a result, for a given heat input to the distillation column, there is an optimum location for the intermediate reboiler that gives the highest thermodynamic efficiency of the distillation column. Similarly there exists an optimum location for an intermediate condenser. The objective of this section is to calculate the optimum location for each case and the resulting maximum thermodynamic efficiency.

In an earlier article we analyzed the thermodynamic efficiency of a conventional binary distillation column with one bottom reboiler and one top condenser (Agrawal and Herron, 1997). The method, along with the associated assumptions described in that article, is extended to analyze the current problem with an intermediate reboiler or an intermediate condenser. The simplifying assumptions made are: ideal vapor phase, ideal liquid solution, equal latent heats for both components A and B, no pressure-drop losses, latent heat independent of temperature (in the operating temperature range of the distillation column), and vapor pressures of components given by the Clausius-Clapeyron equation. In order to calculate the efficiency of the distillation column alone, exergy losses for only the streams entering and leaving the distillation column are considered. Both the bottom and intermediate reboilers and the top and intermediate condensers and any other heat exchangers are excluded from the control volume under consideration. Subcooled liquid or superheated vapor feeds are not analyzed in this study. It is worth pointing out that for a large number of real binary mixtures with near-ideal thermodynamic properties, a good

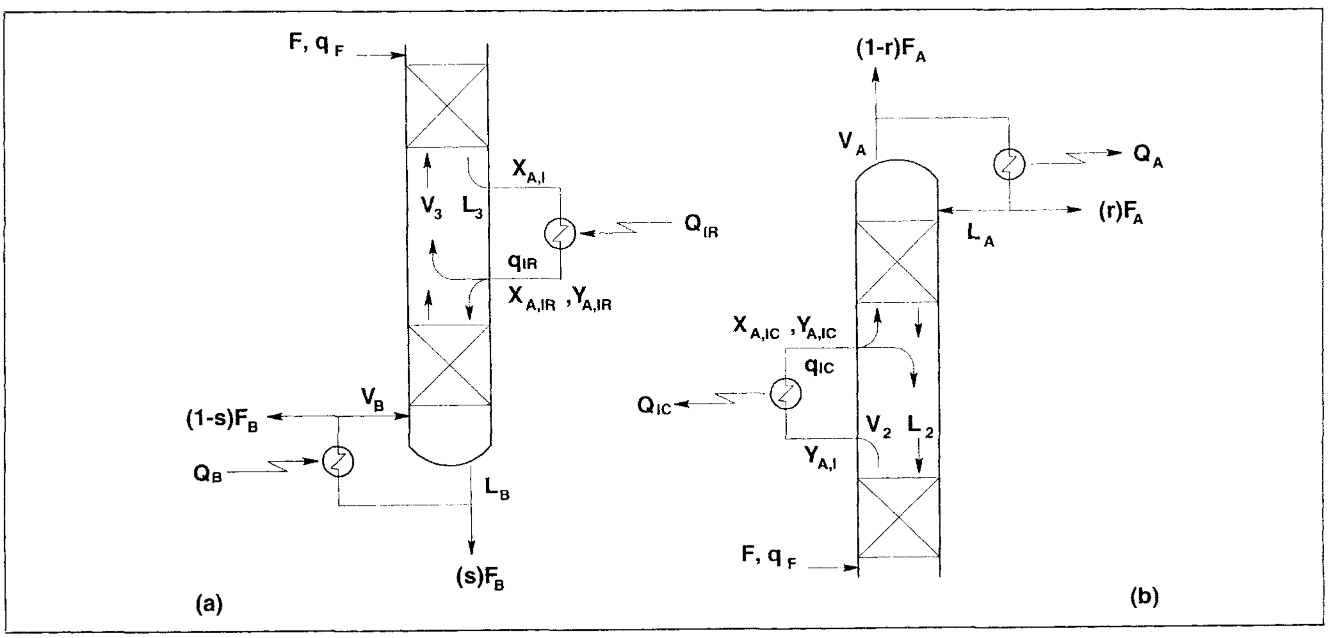

Figure 1. Configurations for side heat duties: (a) intermediate reboiler; (b) intermediate condenser.

agreement was found between the thermodynamic efficiencies calculated from the simplified model and the ones calculated through the computer simulation using a detailed design software with actual thermodynamic properties (Agrawal and Herron, 1997).

Calculations are first presented for the case with an intermediate reboiler, followed by the case with an intermediate condenser. In all cases, a binary mixture AB is distilled to produce pure products A and B. Component A is more volatile than component B.

Intermediate reboiler case

All the liquid of composition $x_{A,I}$ is withdrawn from an intermediate location below the feed to the distillation column and is then partially vaporized in an intermediate reboiler (Figure 1a). The phases leaving the intermediate reboiler are assumed to be in equilibrium. The fraction of liquid in the two-phase stream exiting the intermediate reboiler is q_{IR} and its composition is $(x_{A,IR}, y_{A,IR})$. The McCabe–Theile diagram for this distillation column is shown in Figure 2a (Agrawal and Fidkowski, 1996). Following the method introduced earlier (Agrawal and Herron, 1997) and considering the exergy of streams entering and leaving the distillation column, the expression for the thermodynamic efficiency of the distillation column is derived to be

$$\eta_{IR} = \frac{-RT_o(Z_A \ln Z_A + Z_B \ln Z_B)}{\delta + \Delta HT_o \int_0^{q_f} \left[\frac{1}{T} - \frac{1}{T_A}\right] dq + L_B RT_o \ln \alpha + L_3 \Delta HT_o \int_{q_{IR}}^1 \left[\frac{1}{T_A} - \frac{1}{T}\right] dq}, \quad (1)$$

where

$$\delta = E_F^T - F_A E_A - F_B E_B. \quad (2)$$

In Eq. 1, the term in the numerator is the minimum work of separation and the denominator is the total exergy used by the distillation column to perform the separation. In Eq. 2, the term δ is the difference between the thermal exergies of saturated vapor feed and those of saturated vapors A and B. Generally this difference is small, and in this work δ is taken to be zero. The second term in the denominator of Eq. 1 can be solved for a feed composition by the method described by Agrawal and Herron (1997). The last term in the denominator of Eq. 1 can be calculated from

$$L_3 \Delta HT_o \int_{q_{IR}}^1 \left[\frac{1}{T_A} - \frac{1}{T}\right] dq = - L_3 RT_o \int_{q_{IR}}^1 \ln\left[\frac{x_A(\alpha - 1) + 1}{\alpha}\right] dq, \quad (3)$$

where x_A is the liquid-phase composition as the stream of initial composition $x_{A,I}$ is boiled. As a function of q, x_A is given by

$$x_{A,I} = qx_A + (1-q)\frac{\alpha x_A}{1 + x_A(\alpha - 1)}. \quad (4)$$

For a given overall feed composition Z_A, relative volatility α, and intermediate reboiler liquid feed composition $x_{A,I}$, the calculation of thermodynamic efficiency requires knowl-

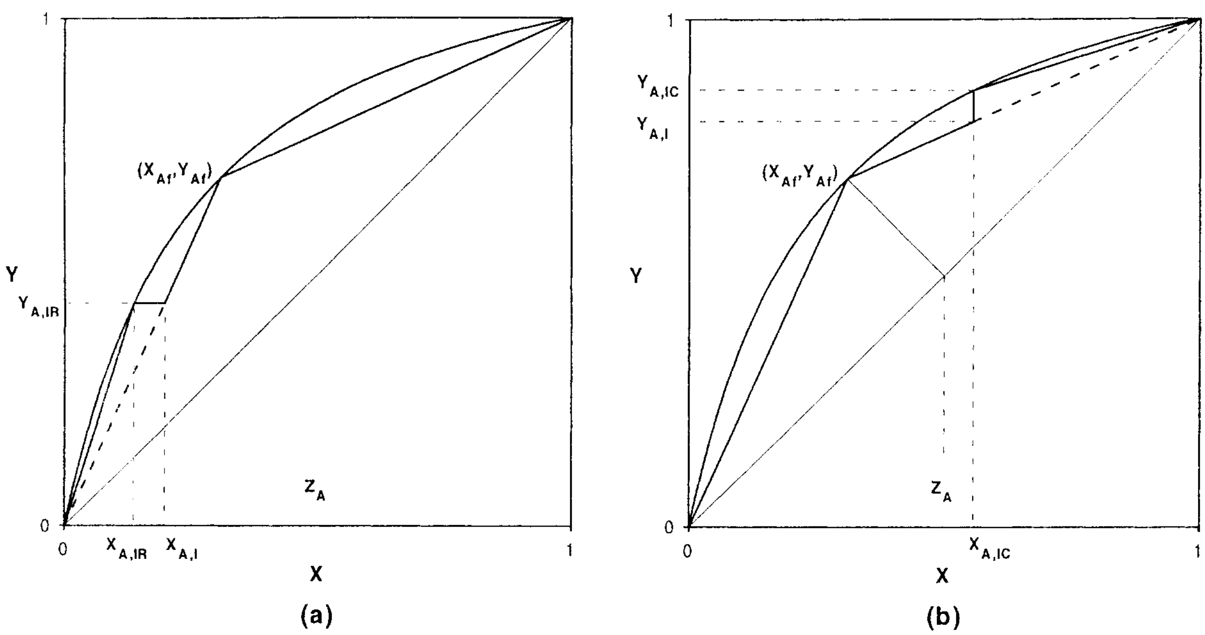

Figure 2. McCabe–Thiele diagrams: (a) intermediate reboiler; (b) intermediate condenser.

edge about the liquid flow in the bottom section (L_B), liquid flow to intermediate reboiler (L_3), and the parameter q_{IR}. In this article, these values are calculated for a pinched distillation column. The McCabe–Thiele diagram shown in Figure 2a is helpful in visualizing the calculation of these variables. The distillation column is taken to be pinched at both the intermediate reboiler and feed locations.

The first step is to calculate the composition $(x_{A,IR}, y_{A,IR})$ of the two-phase stream leaving the intermediate reboiler. From the operating line of the distillation column section bounded by the feed and the location of the intermediate reboiler:

$$y_{A,IR} = x_{A,I}\left(\frac{y_{Af}}{x_{Af}}\right), \tag{5}$$

where the composition of the two-phase feed to the distillation column is given to be (x_{Af}, y_{Af}). Now $x_{A,IR}$ is calculated through the vapor–liquid equilibrium relationship:

$$x_{A,IR} = \frac{y_{A,IR}}{\alpha-(\alpha-1)y_{A,IR}} \tag{6}$$

vapor and liquid flows in the section immediately below the feed can be calculated by mass balance:

$$V_3 = x_{Af}\left[\frac{(1-x_{Af})}{(y_{Af}-x_{Af})}-(1-q_f)\right] \tag{7}$$

$$L_3 = V_3 \cdot \frac{y_{Af}}{x_{Af}}. \tag{8}$$

From the slope of the operating line in the bottom section of the distillation column,

$$L_B = V_B \cdot \frac{y_{A,IR}}{x_{A,IR}}, \tag{9}$$

where V_B can be calculated by the material balance between the two sections below the feed:

$$V_B = \frac{V_3\left(\dfrac{y_{Af}}{x_{Af}}-1\right)}{\left(\dfrac{y_{A,IR}}{x_{A,IR}}-1\right)}. \tag{10}$$

The liquid fraction of the stream leaving the intermediate reboiler is given by

$$q_{IR} = 1-\frac{(V_3-V_B)}{L_3}. \tag{11}$$

Thus, for a given feed condition, relative volatility and composition of the liquid to the intermediate reboiler ($x_{A,I}$), the efficiency of the pinched distillation column can now be calculated from Eq. 1. The liquid flow to intermediate reboiler, L_3, is given by Eqs. 7 and 8. The liquid flow exiting the bottom of the distillation column, L_B, is calculated from Eqs. 5–7, 9 and 10. The fraction of liquid in the stream exiting the intermediate reboiler, q_{IR}, can be calculated from Eqs. 5–7, 10 and 11.

It should be noted that temperatures do not appear explicitly in the calculation of the thermodynamic efficiency. It is sufficient to know feed composition and its quality (q_f), relative volatility, and the composition of the liquid to the intermediate reboiler. This allows us to calculate thermodynamic efficiency by varying each parameter and to draw generalized conclusions. For this purpose, two specific cases are studied in detail: (1) one saturated liquid feed, and (2) one saturated vapor feed. The pertinent equations for each of these cases can be readily derived from Eqs. 1–11.

First consider the case of an intermediate reboiler with *one saturated liquid feed*. For this case $q_f = 1$ and

$$x_{Af} \equiv Z_A \tag{12}$$

$$y_{Af} = \frac{\alpha Z_A}{Z_A(\alpha-1)+1}. \tag{13}$$

After some algebraic manipulation and with the approximation $\delta = 0$, the thermodynamic efficiency from Eq. 1 is

$$\eta_{l,IR} = \frac{-(Z_A \ln Z_A + Z_B \ln Z_B)}{V_B \ln \alpha - L_3 \int_{q_{IR}}^{1} \ln\left[\dfrac{x_A(\alpha-1)+1}{\alpha}\right] dq}, \tag{14}$$

where x_A in the integral of the denominator is calculated from Eq. 4.

Similarly, for the case of an intermediate reboiler with *one saturated vapor feed*, $q_f = 0$, and the pertinent equations are

$$y_{Af} \equiv Z_A \tag{15}$$

$$x_{Af} = \frac{Z_A}{\alpha-(\alpha-1)Z_A}. \tag{16}$$

From Eq. 1, with the approximation that $\delta \approx 0$,

$$\eta_{V,IR} = \frac{-(Z_A \ln Z_A + Z_B \ln Z_B)}{L_B \ln \alpha - L_3 \int_{q_{IR}}^{1} \ln\left[\dfrac{x_A(\alpha-1)+1}{\alpha}\right] dq}. \tag{17}$$

The maximum thermodynamic efficiency for a given feed composition was determined numerically for each of these cases by calculating the optimal value of $x_{A,I}$. For a given feed composition, the values of maximum thermodynamic efficiency for the two cases are plotted as a function of relative volatility α in Figures 3 and 4. For reference purposes, thermodynamic efficiency curves without the intermediate reboiler are also plotted in these figures. As expected, for any given feed, the use of an intermediate reboiler always leads to improvement in efficiency.

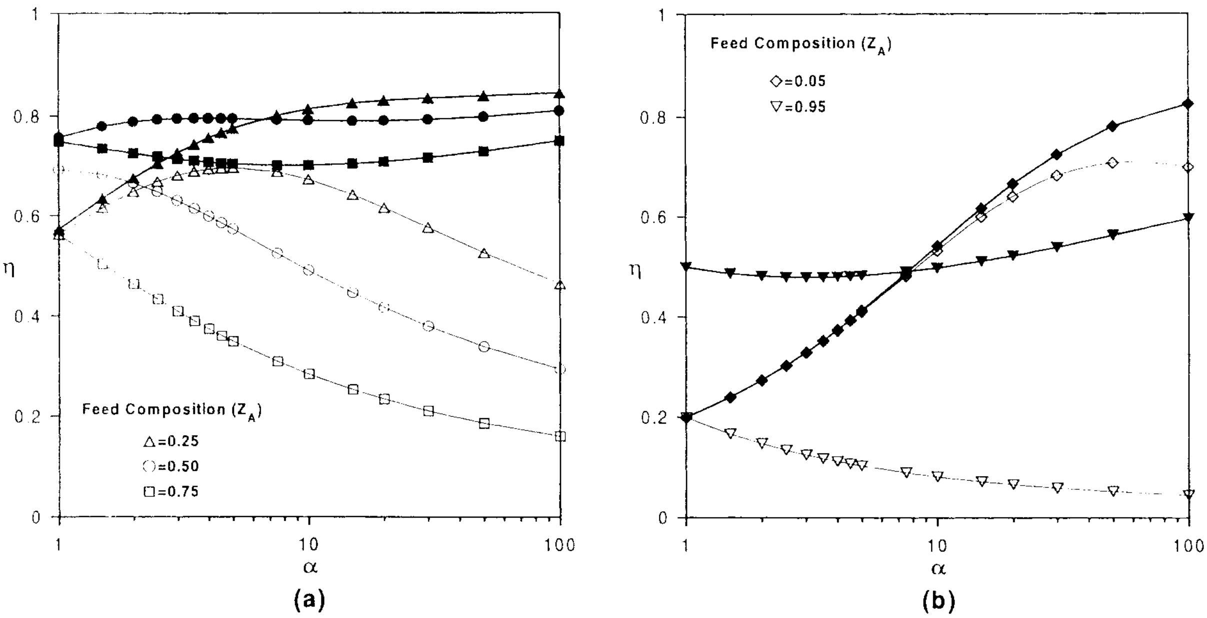

Figure 3. Maximum thermodynamic efficiency as a function of α for a saturated liquid feed.

Dark lines (filled symbols) are for a column with an intermediate reboiler; light lines (open symbols) are for a column without an intermediate reboiler.

Intermediate condenser case

All the vapor of composition $y_{A,I}$ is withdrawn from an intermediate location above the feed to the distillation column and is sent to an intermediate condenser (Figure 1b). The partially condensed stream from this condenser is returned to the same location in the distillation column. The McCabe–Thiele diagram for a pinched column with an intermediate condenser is shown in Figure 2b. The equations used to calculate thermodynamic efficiency for this case can be derived by following the steps described for the intermediate reboiler case and are summarized in the Appendix.

Similar to the intermediate reboiler cases, the maximum thermodynamic efficiency with an intermediate condenser for a prespecified feed condition was calculated numerically at the optimal value of $y_{A,I}$. The maximum thermodynamic efficiencies for fixed feed compositions as a function of relative volatility α are given in Figure 5 for the saturated liquid-feed case, and in Figure 6 for the saturated vapor-feed case. In

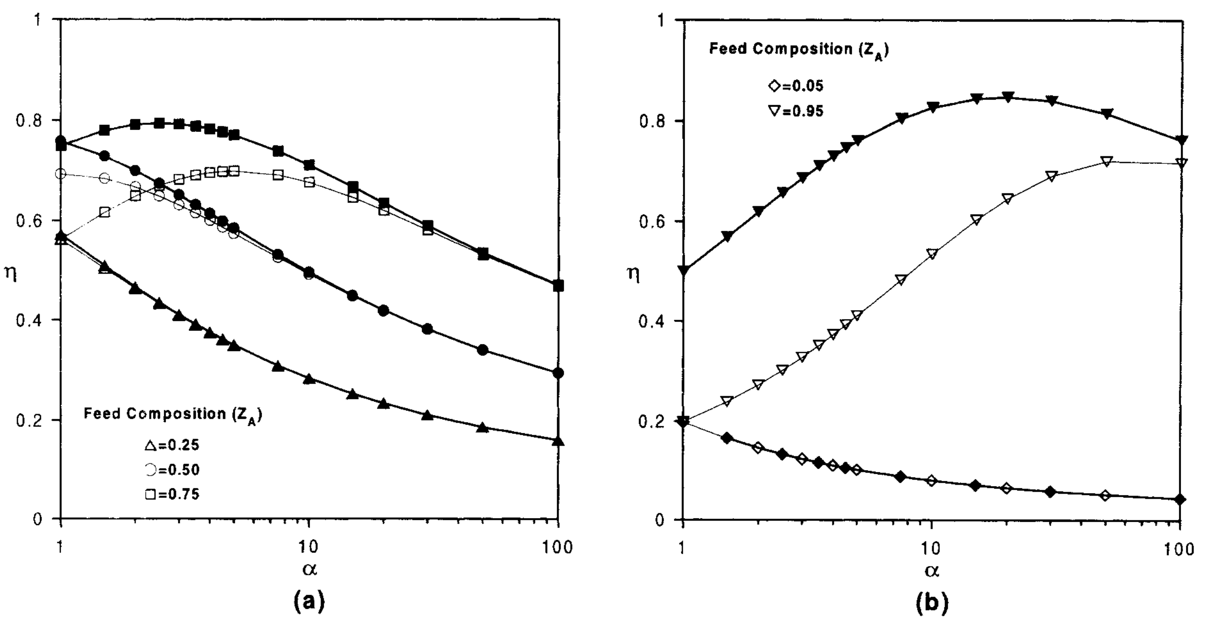

Figure 4. Maximum thermodynamic efficiency as a function of α for a saturated vapor feed.

Dark lines (filled symbols) are for a column with an intermediate reboiler; light lines (open symbols) are for a column without an intermediate reboiler.

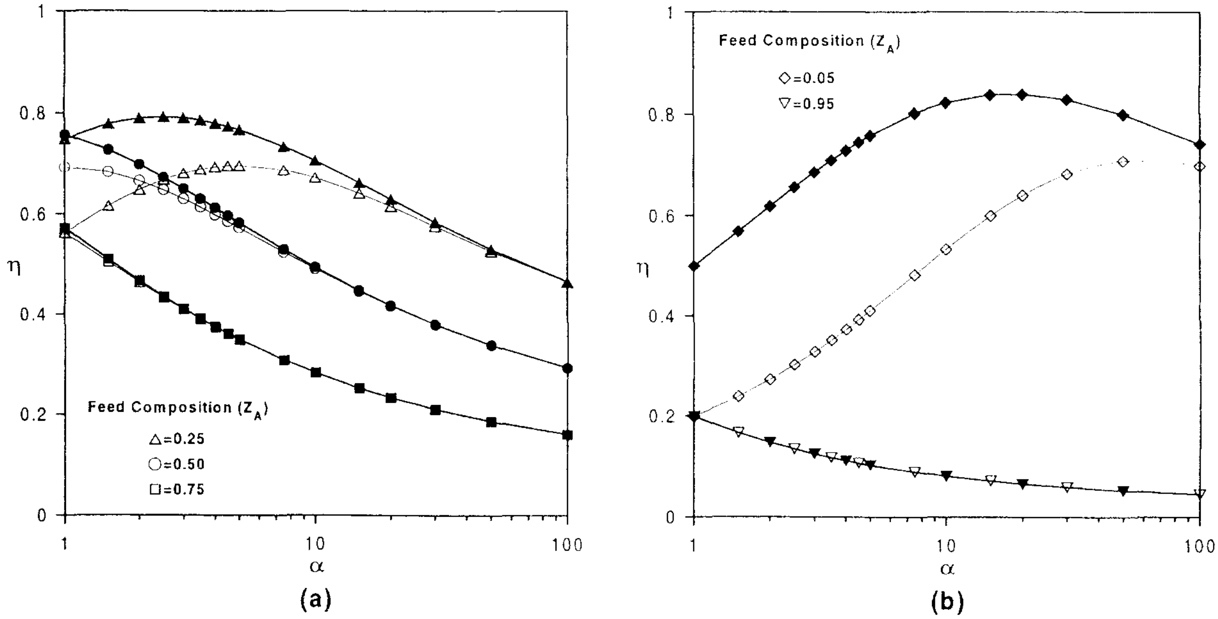

Figure 5. Maximum thermodynamic efficiency as a function of α for a saturated liquid feed.

Dark lines (filled symbols) are for a column with an intermediate condenser; light lines (open symbols) are for a column without an intermediate condenser.

both these figures, for reference purposes, thermodynamic efficiency curves for the base distillation column without any intermediate condenser are also plotted. As expected, the use of an intermediate condenser always improves the efficiency of the distillation.

Discussion

In this section an attempt is made to answer questions raised in the introductory section regarding the efficient use of an intermediate reboiler or an intermediate condenser. The questions to be answered for a given saturated vapor or saturated liquid feed are: When is an intermediate reboiler or an intermediate condenser effective or ineffective in significantly increasing the efficiency of distillation? When is an intermediate reboiler more effective than an intermediate condenser or vice versa? These two questions are answered first for the all-liquid-feed case and then for the all-vapor-feed case. While it is difficult to have a general definition of significant increase in efficiency, in this work an improvement of

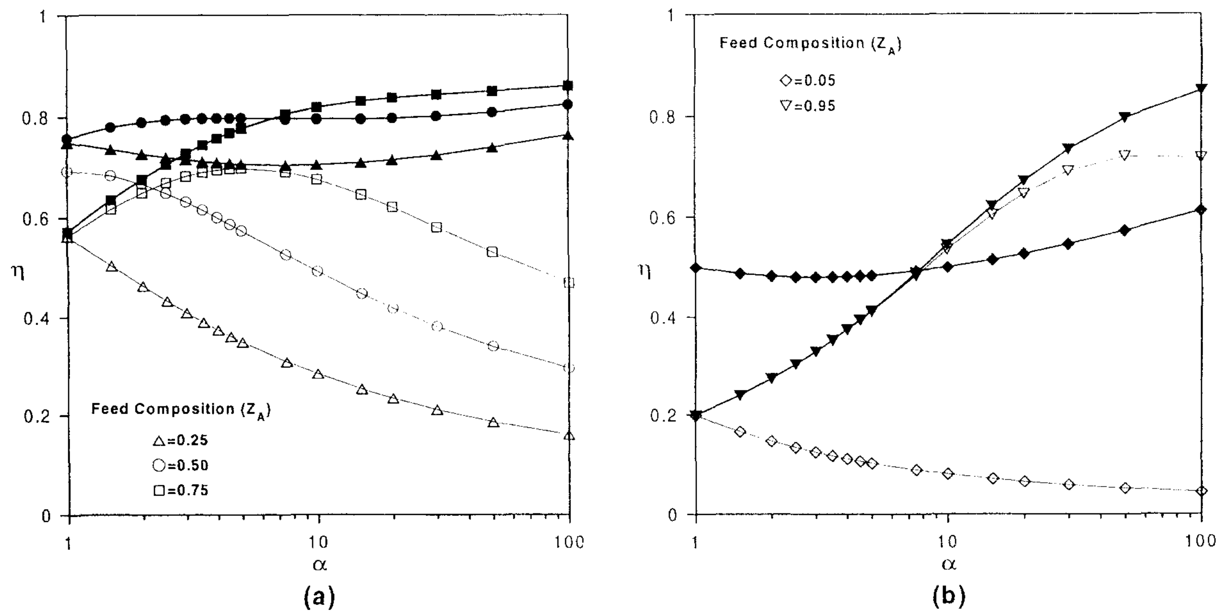

Figure 6. Maximum thermodynamic efficiency as a function of α for a saturated vapor feed.

Dark lines (filled symbols) are for a column with an intermediate condenser; light lines (open symbols) are for a column without an intermediate condenser.

more than about 3% was taken to mean a significant improvement in efficiency.

Observations for the all-liquid-feed case

For this case of one saturated liquid feed, observations from the calculated results are first presented to help in the understanding of the optimal use of an intermediate reboiler or an intermediate condenser (Figures 3 and 5):

1. For a liquid feed with $Z_A \geq 50\%$ (feed rich in light component A), the efficiency of the base distillation (no intermediate reboiler or condenser) declines as α increases. This trend remains unchanged with the use of an intermediate condenser.
2. For $Z_A \geq 50\%$, an intermediate condenser shows only marginal improvement in efficiency. The improvement in efficiency is only observed at low values of α and it quickly diminishes as the value of α is increased (say to α above 3).
3. For $Z_A \geq 50\%$, an intermediate reboiler always produces a greater efficiency improvement than an intermediate condenser.
4. For $Z_A \geq 50\%$, the efficiency curve for the intermediate reboiler shows a mild minimum in efficiency with increasing values of α (with the exception of data in the neighborhood of $Z_A = 50\%$ and α close to one). Even with this trend, the improvement in efficiency as compared to the base distillation is quite substantial. This gain in efficiency is even more remarkable when Z_A is greater than 90%, that is, feed is very rich in light component A. For example, when $Z_A = 95\%$ and $\alpha = 5$ (Figure 3b), the efficiency for base distillation is 10.3%, while that with an intermediate reboiler is 48.3%.
5. For $Z_A = 50\%$, with an intermediate reboiler, efficiency is a weak function of α. However, the improvement in efficiency as compared to the base case or to the corresponding intermediate condenser case is quite substantial.
6. For $Z_A < 50\%$, the efficiency with an intermediate reboiler always increases with increasing values of α. For such feed compositions at lower values of α, however, an intermediate condenser can give higher efficiencies than an intermediate reboiler.
7. For $Z_A < 50\%$, an intermediate condenser is more effective than an intermediate reboiler at lower values of α. For a given feed composition of $Z_A < 50\%$, however, there exists a value of α beyond which the efficiency with an intermediate condenser starts to decline, and eventually the efficiency with an intermediate reboiler is greater. This value of α increases as Z_A approaches zero.
8. For feeds very rich in B ($Z_A \leq 10\%$) the improvement in efficiency at low to modest values of α due to an intermediate reboiler is negligible. For such cases, however, the improvement in efficiency due to an intermediate condenser is quite impressive. For example, at $Z_A = 5\%$ and $\alpha = 3$, the efficiency of base distillation is 32.8%, while with an intermediate condenser it is 68.6%. This improvement is even more impressive as the value of Z_A is further decreased; at $Z_A =$ 1% and $\alpha = 5$, the efficiency with an intermediate condenser is 56% as compared to 13.4% for the base distillation.
9. A peculiar observation from inspection of numbers generated through calculations is that for a given feed composition, the optimum location of the intermediate condenser is such that the composition of the liquid phase in the two-phase stream exiting the intermediate condenser, $x_{A,IC}$, is independent of α and is only a function of the feed composition. The optimal values of $x_{A,IC}$ as a function of feed composition is shown in Figure 7. No attempt was made to explain this observation through equations. No such simple observation could be drawn for the intermediate reboiler case.
10. Another peculiar observation made from the calculated results is that the fraction of the total liquid flow in the section immediately above the feed that is produced from the intermediate condenser is approximately (if not exactly) equal to $1 - x_{A,IC}$, and its value is independent of α. For example, when $Z_A = 0.6$, the value of $x_{A,IC}$ is 0.775 and the fraction of total liquid in the section immediately above the feed that comes from the intermediate condenser is 0.226.
11. Another curious observation is that the fraction of vapor in the section immediately above the intermediate reboiler that is produced from the intermediate reboiler is found to be approximately (if not exactly) equal to $y_{A,IR}$. For example, at $Z_A = 0.25$ and $\alpha = 2.5$, the value of $y_{A,IR}$ is 0.261 and the fraction of the total vapor in the section above the intermediate reboiler that comes from the intermediate reboiler is found to be 0.262. However, for a given feed composition, the optimum value of $y_{A,IR}$ does vary with α.

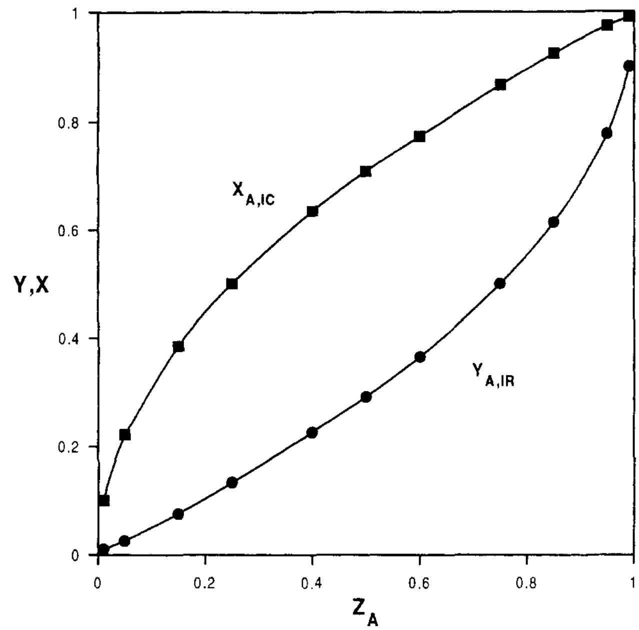

Figure 7. Optimum values of $x_{A,IC}$ for the liquid-feed case (squares), and $y_{A,IR}$ for the vapor-feed case (circles).

Efficiency-based heuristics for the all-liquid-feed case

Observations 1 through 8 provide the much needed clues to help in deriving generalized heuristics regarding the effectiveness of an intermediate reboiler or an intermediate condenser for a saturated liquid-feed case. It turns out that another needed piece of information is the optimum quality of feed (q_{opt}), which gives the maximum thermodynamic efficiency when no intermediate reboiler nor intermediate condenser is used. The values of q_{opt} as a function of Z_A for several values of α are reproduced in Figure 8 (Agrawal and

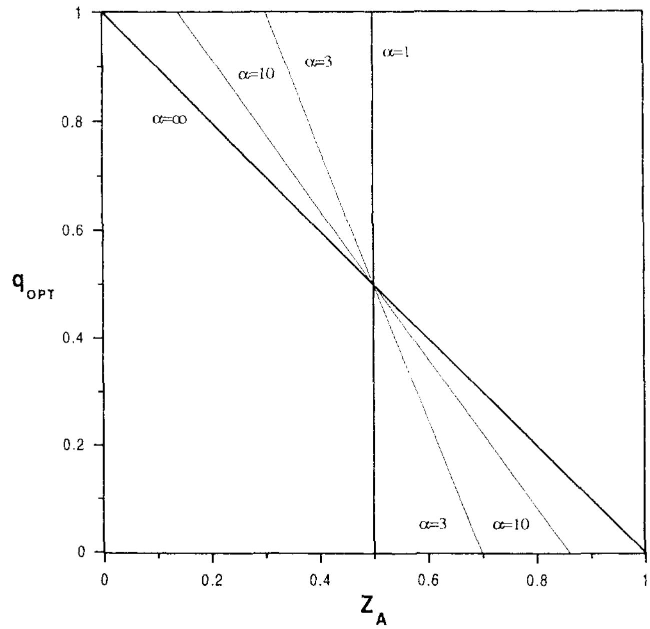

Figure 8. Optimum liquid fraction q_{opt} for a single feed with no intermediate reboiler or intermediate condenser.

Operation at the designated liquid fraction yields maximum thermodynamic efficiency.

Herron, 1997). For a given feed composition and relative volatility, the value of q_{opt} can be read from this figure. If q_{opt} is less than 1, then the preferred feed state for the base distillation column (without intermediate reboiler or condenser) is either two-phase or all-vapor. In such cases, if a saturated liquid feed were employed, an intermediate reboiler would be effective. The lower the value of q_{opt}, the more effective an intermediate reboiler will be in improving the efficiency of the base distillation column. It is as if some vapor is needed in the feed of the base distillation column, and since the feed is saturated liquid, column efficiency can be substantially improved by using an intermediate reboiler. Furthermore, for q_{opt} values less than about 0.85, an intermediate condenser provides only a marginal improvement in efficiency. For such cases, the optimum location of the intermediate condenser is very close to the top condenser, and therefore the efficiency gain due to a temperature difference between the two condensers is negligible. Basically, the temperature difference between the cold utility needed for the top condenser and that needed for the intermediate condenser is small.

When q_{opt} is close to one, an intermediate condenser starts to become more effective. For $q_{opt} > 0.85$, an intermediate condenser can lead to a meaningful improvement in efficiency, and when $q_{opt} = 1$, substantial improvements in efficiency can be achieved with an intermediate condenser.

A parameter that can help in determining the effectiveness of an intermediate condenser is Z_l^*. For a given α, the maximum value of Z_A that would require q_{opt} to be one is defined as Z_l^*. For example, from Figure 8 the value of Z_l^* is 0.3 for $\alpha = 3$. For a given α, the value of Z_l^* can be read from Figure 8 or it can be calculated from Eqs. 37 and 38 of Agrawal and Herron (1997). If the actual feed composition Z_A is equal to Z_l^*, an intermediate condenser and an intermediate reboiler are equally effective. For Z_A less than Z_l^*, an intermediate condenser is more effective than an intermediate reboiler. As the difference $(Z_l^* - Z_A)$ increases, an intermediate reboiler becomes increasingly less effective and the improvement obtained from an intermediate condenser rises. All these observations for a given liquid feed composition and relative volatility are summarized in Table 1.

Information given in Table 1 requires knowledge of q_{opt}. This means that either Figure 8 or the approximate equations for q_{opt} discussed in the earlier article must be readily available while making the decision about the effectiveness of an intermediate reboiler or an intermediate condenser. Fortunately, approximate and quick guidance can be obtained by using the parameters x_A^* and y_A^* from the earlier article; x_A^* and y_A^* are easy to locate on a McCabe-Thiele diagram, as they are the coordinates for the point of intersection of the equilibrium curve and inverse diagonal ($y = 1 - x$). For the case of distillation with no intermediate condenser or reboiler, for some Z_A slightly less than x_A^*, the preferred feed state is all-liquid; and similarly for some Z_A slightly greater than y_A^*, the preferred feed state is all-vapor. For $x_A^* \le Z_A \le y_A^*$, the preferred state is definitely two-phase. For a given liquid feed, the value of x_A^* is either read from a McCabe-Thiele diagram or can be readily calculated from the following equation:

$$x_A^* = \frac{1}{\sqrt{\alpha} + 1}. \tag{18}$$

For all values of $Z_A \ge x_A^*$, an intermediate reboiler will give greater improvement in efficiency than an intermediate condenser. For $Z_A \ll x_A^*$, an intermediate condenser will be much more effective. Pertinent results are summarized in Table 2.

As an example on how to use Table 2, consider a saturated liquid feed containing 24% A with a relative volatility α of 10. The McCabe-Thiele diagram for this case is shown in Figure 9. The value of x_A^* for this case is the same as the feed composition. The operating lines for both stripping and rectifying sections are symmetric about the inverse diagonal. From inspection of the McCabe-Thiele diagram alone it is difficult to assess which of the two intermediate duty choices

Table 1. Efficiency-Based Guidelines for Selection Between an Intermediate Reboiler or Condenser for a Saturated Liquid Feed (Based on q_{opt})

Case	Comment
$q_{opt} < 0.85$	Intermediate reboiler is very effective; intermediate condenser of marginal value
$0.85 < q_{opt} < 1$	Intermediate reboiler still provides more efficiency improvement, but intermediate condenser can be useful
$q_{opt} = 1$ and $Z_A = Z_l^*$	Both intermediate reboiler and intermediate condenser are equally effective
$q_{opt} = 1$ and $Z_A < Z_l^*$	Intermediate condenser is more effective. As the difference $Z_l^* - Z_A$ increases, improvement from an intermediate reboiler becomes marginal, but improvement from an intermediate condenser becomes substantial

Table 2. Efficiency-Based Guidelines for Selection Between an Intermediate Reboiler or Condenser for a Saturated Liquid Feed (Based on x_A^*)

Case	Comment
$Z_A \gg x_A^*$	Marginal impact of intermediate condenser; high impact of intermediate reboiler
$Z_A = x_A^*$	Intermediate reboiler more efficient than intermediate condenser
$Z_A < x_A^*$	As $(x_A^* - Z_A)$ increases, impact of intermediate condenser becomes comparable to intermediate reboiler and eventually exceeds it
$Z_A \ll x_A^*$	High impact of intermediate condenser; marginal impact of intermediate reboiler

would give more improvement in efficiency. However, the guidelines of Table 2 recommend that an intermediate reboiler would be more effective than an intermediate condenser. Indeed calculations show that the efficiency of the base distillation (all-liquid feed) is 68%. An intermediate condenser only improves it to 72%, while an intermediate reboiler improves it to 82%. The reason is that when $Z_A = x_A^*$, the preferred feed state for distillation still consists of some vapor ($q_{opt} = 0.87$, q_{opt} is approaching 1 but is slightly less than 1), and an intermediate reboiler is more effective by providing the needed vapor.

Observations for the all-vapor-feed case

For the case of one saturated vapor feed, questions regarding the utility of an intermediate reboiler or an intermediate condenser can be answered through observations from Figures 4 and 6 and the other calculated results. This treatment is similar to the one described for the saturated liquid-feed case.

1. For a vapor feed with $Z_A \leq 50\%$ (feed rich in heavy component B), the efficiency of the base distillation declines with increasing α. This trend is unchanged when an intermediate reboiler is used.
2. For $Z_A \leq 50\%$, an intermediate reboiler shows only marginal improvement in efficiency for all values of α. Furthermore, this improvement declines rapidly with increasing values of α, and for α greater than approximately 3 the efficiency incentive for an intermediate reboiler is small. Basically, for such cases the optimum location of the intermediate reboiler is too close to the bottom reboiler.
3. For $Z_A \leq 50\%$, an intermediate condenser always yields higher efficiency than an intermediate reboiler.
4. For $Z_A \leq 50\%$ and for values of α slightly greater than one, an intermediate condenser shows a mild minimum in efficiency with further increasing values of α. In spite of this trend, the improvement in efficiency compared to the base distillation column is fairly large for all values of α. Particularly at very low values of Z_A (less than 10%, that is, feed is nearly B), the improvement in efficiency due to an intermediate condenser is extremely large.
5. For $Z_A = 50\%$, with an intermediate condenser, efficiency is a weak function of α. However, the improvement in efficiency as compared to an intermediate reboiler case or to the base distillation case is substantial.
6. For $Z_A > 50\%$, the efficiency with an intermediate condenser always increases with increasing values of α. However, at lower values of α, for such feed compositions, an intermediate reboiler can give higher efficiencies than can an intermediate condenser.
7. For $Z_A > 50\%$, an intermediate reboiler is more effective at lower values of α. For a given feed composition with $Z_A > 50\%$, there exists a value of α beyond which the efficiency with an intermediate reboiler starts to decline. This value of α increases as Z_A approaches 100%.
8. For feeds very rich in A ($Z_A \geq 90\%$), the improvement in efficiency at low to modest values of α due to an intermediate condenser is negligible. In such situations, however, the improvement in efficiency due to an intermediate reboiler is remarkable. For example, at $Z_A = 99\%$ and $\alpha = 5$, the efficiency of the base distillation is 13.4%, while that for an intermediate reboiler is 56.1%! This tells us that when distilling a gaseous feed mixture very rich in the volatile component, it may be desirable to use an intermediate reboiler.

Curious observations analogous to 9 through 11 for the saturated liquid-feed case also exist for the saturated vapor-feed case. For example, for a given feed composition the optimum location of the intermediate reboiler is such that the mole fraction of A in the vapor phase of the two-phase stream exiting the intermediate reboiler, $y_{A,IR}$, is independent of α and is only a function of the feed composition. Figure 7 shows the optimum values of $y_{A,IR}$ as a function of feed composition Z_A. Once again, no attempt is made to explain these observations.

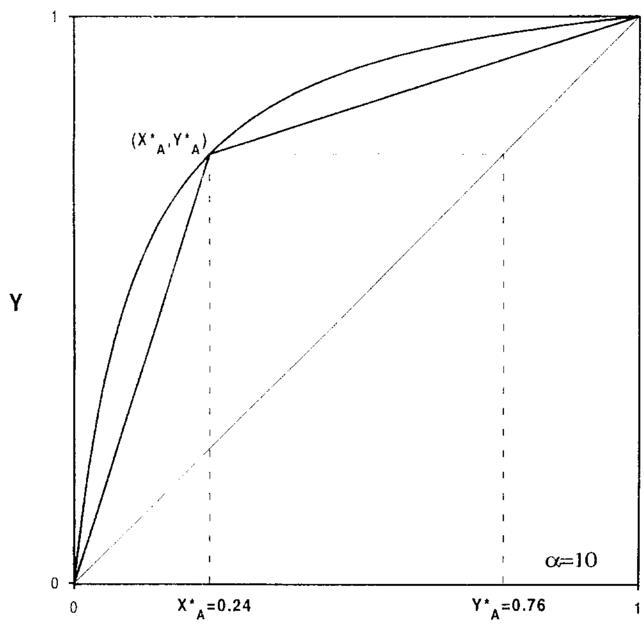

Figure 9. McCabe–Thiele diagram for either a saturated liquid feed containing 24% A or a saturated vapor feed containing 76% A.
The relative volatility, α, is 10.

Efficiency-based heuristics for the all-vapor-feed case

From observations 1 through 8, heuristics analogous to those obtained for the liquid-feed case can be derived and

Table 3. Efficiency-Based Guidelines for Selection Between an Intermediate Reboiler or Condenser for a Saturated Vapor Feed (Based on q_{opt})

Case	Comment
$q_{opt} = 0$ and $Z_A > Z_v^*$	Intermediate reboiler is more effective. As the difference $Z_A - Z_v^*$ increases, improvement from an intermediate condenser becomes marginal and that from an intermediate reboiler substantial
$q_{opt} = 0$ and $Z_A = Z_v^*$	Both intermediate reboiler and intermediate condenser are equally effective
$0 < q_{opt} < 0.15$	Intermediate reboiler is still effective, but intermediate condenser is more effective
$0.15 < q_{opt}$	Intermediate condenser is very effective; intermediate reboiler is only marginally effective

are summarized in Tables 3 and 4. In Table 3, Z_v^* is defined as the lowest value of Z_A, at a given value of α, for which $q_{opt} = 0$. For example, when $\alpha = 3$ the value of Z_v^* from Figure 8 is 0.7. For all values of Z_A greater than Z_v^* the preferred feed state for the base distillation column (with no intermediate reboiler or intermediate condenser) is all-vapor. Basically, Table 3 sums up the pivotal observation that for a saturated vapor-feed case, if the preferred feed state for the base distillation is all-vapor, then an intermediate reboiler is more effective, and if the preferred feed state is two-phase or all-liquid, then an intermediate condenser is more effective. Also, if the amount of liquid in the preferred feed state for the base distillation is greater than 10 to 15%, then the improvement in efficiency due to an intermediate reboiler would be marginal. For a given saturated vapor feed composition and relative volatility, the value of q_{opt} and Z_v^* can be read from Figure 8 and a decision regarding the efficient use of an intermediate reboiler or a condenser can be readily made from Table 3.

When Figure 8, or the equations that approximate the q_{opt} lines, are not readily available, an approximate but fairly good decision can be reached by using y_A^*, described earlier, and Table 4. The value of y_A^* can be read from a McCabe–Thiele diagram or it can be calculated from the following equation:

Table 4. Efficiency-Based Guidelines for Selection Between an Intermediate Reboiler or Condenser for a Saturated Vapor Feed (Based on y_A^*)

Case	Comment
$Z_A \ll y_A^*$	Marginal impact of intermediate reboiler; high impact of intermediate condenser
$Z_A = y_A^*$	Intermediate condenser more efficient than intermediate reboiler
$Z_A > y_A^*$	As $(Z_A - y_A^*)$ increases, impact of intermediate reboiler becomes comparable to intermediate condenser and eventually exceeds it
$Z_A \gg y_A^*$	High impact of intermediate reboiler; marginal impact of intermediate condenser

$$y_A^* = 1 - x_A^*, \qquad (19)$$

where x_A^* is given by Eq. 18. Once again, the McCabe–Thiele diagram in Figure 9 can be used to illustrate the use of Table 4. Consider a saturated vapor feed containing 76% A with relative volatility α of 10. The value of Z_A is equal to y_A^*, and from Table 4 we expect that an intermediate condenser would be more effective than an intermediate reboiler. Calculations show that the efficiency of the base distillation column is 68%. Use of an intermediate reboiler improves it to 72%, but an intermediate condenser is more effective by improving it to 82%.

Efficient choice for a two-phase feed case

Thermodynamic efficiency calculations were also performed for a two-phase feed. For these calculations the liquid fraction of the feed was chosen such that it was equal to mole fraction of heavy component B in the feed ($q_f = 1 - Z_A$). The base distillation case for such a feed condition is discussed in detail elsewhere (Agrawal and Herron, 1997). The improvement in efficiency with an intermediate reboiler or condenser for such a two-phase feed can be seen from the results plotted in Figures 10 and 11. For all the calculated data ($\alpha \leq 100$), the efficiency increases with increasing α for both the intermediate reboiler and condenser cases.

Since $q_f = 1 - Z_A$, for all finite values of α and $Z_A < 50\%$, the liquid fraction of the feed q_f is less than q_{opt}, that is, the fraction of liquid in the feed is less than that which would give the highest efficiency for the base distillation case (Figure 8). For all feeds rich in B ($Z_A < 50\%$), an intermediate condenser is observed to give a greater improvement in efficiency than an intermediate reboiler. Similarly for $Z_A > 50\%$ and finite values of α, the liquid fraction of the feed q_f is greater than q_{opt}, that is, the fraction of vapor in the feed is less than the fraction that would give highest efficiency for the base distillation. It is observed that for such A-rich ($Z_A > 50\%$) feeds, an intermediate reboiler is more effective in improving the efficiency than is an intermediate condenser. When $Z_A = 50\%$, $q_f = q_{opt}$, and since both rectifying and stripping sections are symmetric around the feed point (for example, as seen on a McCabe–Thiele diagram), an intermediate reboiler or an intermediate condenser is equally effective. All these observations regarding the effectiveness of an intermediate reboiler or condenser at various feed compositions for this two-phase feed case are in agreement with observations made earlier for saturated liquid- and saturated vapor-feed cases.

One final observation worthy of attention is that at low to moderate values of α (less than about 7.5) and $Z_A = 50\%$, an all-vapor feed with an intermediate condenser or an all-liquid feed with an intermediate reboiler gives slightly higher efficiency than the two-phase feed ($q_f = q_{opt} = 0.5$) with an intermediate reboiler or condenser. This provides another result—when an intermediate reboiler or condenser is to be used, the optimum feed quality is not exactly the same as that for the base distillation column that does not use an intermediate reboiler or condenser. Another conclusion from this observation is that if the given feed was all-vapor or -liquid, and an intermediate reboiler or condenser were to be used, then the gain in efficiency by preadjusting the liquid fraction of

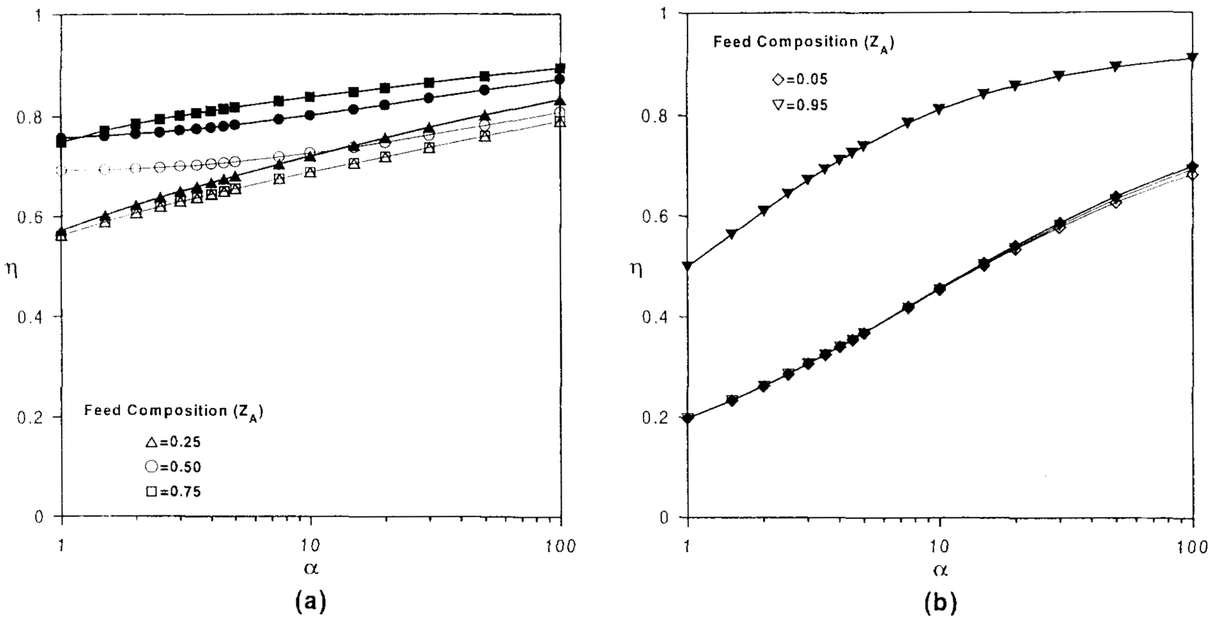

Figure 10. Maximum thermodynamic efficiency as a function of α for a two-phase feed ($q_f = 1 - Z_A$).

Dark lines (filled symbols) are for a column with an intermediate reboiler; light lines (open symbols) are for a column without an intermediate reboiler.

the feed may not be significant. Therefore, if proper utilities were available, it would not be attractive to incur the extra cost of heat exchangers to adjust the quality of the feed.

Conclusions

In this article a simplified model is used to study the effect of an intermediate reboiler or an intermediate condenser on the thermodynamic efficiency of ideal binary mixture distillations producing pure products. It is found that, due to simplifying assumptions, the information needed to calculate thermodynamic efficiencies is simply feed composition, liquid fraction in the feed (q), relative volatility α, and location of the intermediate reboiler or condenser. Temperatures of reboilers and condensers do not explicitly appear in the efficiency equations.

Thermodynamic efficiencies were calculated for a wide range of feed composition and relative volatilities. Maximum

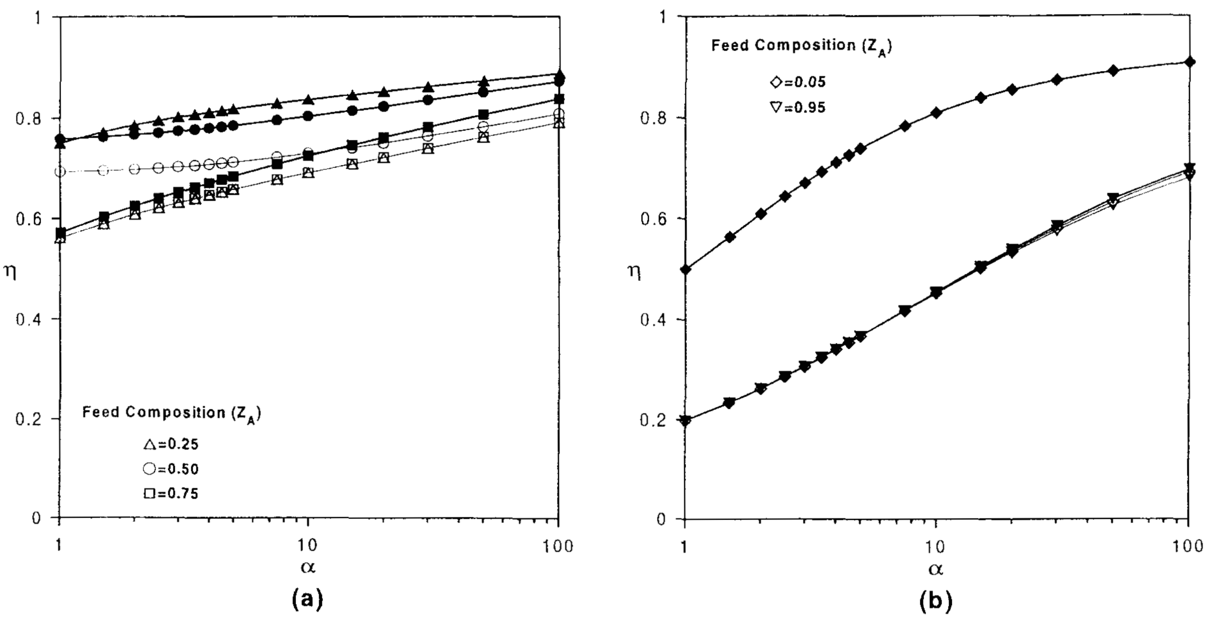

Figure 11. Maximum thermodynamic efficiency as a function of α for a two-phase feed ($q_f = 1 - Z_A$).

Dark lines (filled symbols) are for a column with an intermediate condenser; light lines (open symbols) are for a column without an intermediate condenser.

thermodynamic efficiency was calculated allowing pinches to occur at the feed location and at the intermediate reboiler or condenser location. The position of the intermediate reboiler or condenser was then adjusted to give maximum possible efficiency as a function of feed composition and relative volatility. Some generalized conclusions regarding the effect of an intermediate reboiler or condenser on the efficiency of a distillation column were drawn.

The advantage of the generalizations derived from the calculated results is that they can quickly tell a process engineer if an intermediate reboiler or condenser is going to be effective in improving the efficiency and, out of the two options, which one would be more effective. It is found that a quick and reliable answer can be reached by following the steps described below:

1. For a given feed composition (Z_A) and relative volatility (α), determine the optimum quality (q_{opt}) of the feed that would give maximum thermodynamic efficiency for distillation with no intermediate reboiler or condenser.
2. If the optimum feed quality requires more vapor than that present in the feed ($q_{\text{opt}} < q_f$), then an intermediate reboiler is found to be more effective in improving the efficiency.
3. If the optimum feed quality requires that additional liquid be present in the feed ($q_{\text{opt}} > q_f$), then an intermediate condenser is found to be more effective in improving the efficiency.
4. If the feed contains a lot more vapor than that suggested by the optimum feed quality, then the improvement in the thermodynamic efficiency due to an intermediate reboiler is only marginal. For example, for a given saturated vapor feed ($q_f = 0$), if the optimum feed quality is such that the suggested amount of liquid in the feed is greater than 15% ($q_{\text{opt}} > 0.15$), then an intermediate reboiler is of marginal value in improving thermodynamic efficiency.
5. If the feed contains a lot more liquid than that suggested by the optimum feed quality, then the improvement in the thermodynamic efficiency due to an intermediate condenser is only marginal. For example, for a given saturated liquid feed ($q_f = 1$), if the optimum feed quality is such that the suggested amount of liquid in the feed is less than 85% ($q_{\text{opt}} < 0.85$), then an intermediate condenser only provides marginal improvement in efficiency.

The values of q_{opt} can either be read from Figure 8 or easily calculated through equations described in Agrawal and Herron (1997). Tables 1 and 3 provide a more detailed guideline. Two corollaries of the above steps are:

- For a saturated liquid feed rich in more volatile component A ($Z_A \geq 50\%$), an intermediate condenser will show only a marginal improvement in efficiency, while an intermediate reboiler would be quite effective.
- For a saturated vapor feed rich in heavy component B ($Z_A \leq 50\%$), an intermediate reboiler will yield only marginal improvement in efficiency, while an intermediate condenser would be quite effective.

An approximate but fairly good answer can also be obtained by using the coordinates (x_A^*, y_A^*) of the point of intersection of the equilibrium curve and inverse diagonal on the McCabe–Thiele diagram along with the information in Table 2 or 4.

Generally, when relatively pure streams (Z_A either less than 10% or more than 90%) are distilled, the efficiency of a distillation column is quite low. In such situations a properly chosen intermediate reboiler or condenser can lead to remarkable improvement in efficiency. For a feed very rich in the volatile component A ($Z_A > 90\%$), an intermediate reboiler, and for a feed very rich in the heavy component B ($Z_A < 10\%$), an intermediate condenser, can be used to produce large improvements in thermodynamic efficiency.

The primary benefit of this study is that it provides a quick method to narrow distillation options. If, on the one hand, an intermediate reboiler or condenser provides only marginal improvement in the thermodynamic efficiency of distillation, then capital cost and complexity considerations would rarely justify its use. On the other hand, a substantial improvement in efficiency provides an incentive to search for the proper utilities needed for the intermediate reboiler or condenser. In particular, such information is most useful for subambient distillation or distillations with heat pumps where mechanical work rather than heat energy is used to drive the distillation column.

Notation

E_A, E_B = molar exergy of saturated vapor for A and B
E_F^T = thermal component of exergy of feed at the dewpoint temperature
F_A, F_B = flow rate of products A and B
ΔH = molar latent heat of vaporization (taken as a positive number)
q_{IC} = fraction of liquid in stream exiting intermediate condenser
r = fraction of product A produced as liquid
R = universal gas constant
s = fraction of product B produced as liquid
T_A = condensation temperature of A at distillation column pressure
T_B = condensation temperature of B at distillation column pressure
T_o = reference temperature
V = molar vapor flow rate
$x_{A,I}$ = mole fraction of A in the liquid to the intermediate reboiler
Z_B = overall mole fraction of B in the feed to distillation column

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Appendix: Thermodynamic Efficiency Equation for an Intermediate Condenser Case

The thermodynamic equation for this case is derived to be

$$\eta_{IC} = \frac{-RT_o(Z_A \ln Z_A + Z_B \ln Z_B)}{\delta + \Delta HT_o \int_0^{q_f} \left[\frac{1}{T} - \frac{1}{T_A}\right] dq + (L_A + q_f) RT_o \ln \alpha + V_2 \Delta HT_o \int_0^{q_{IC}} \left[\frac{1}{T} - \frac{1}{T_B}\right] dq}, \quad \text{(A1)}$$

where δ is given by Eq. 2, and q_{IC} is the fraction of liquid in the two-phase stream exiting the intermediate condenser. The last term in the denominator of the preceding equation is calculated from

$$V_2 \Delta HT_o \int_0^{q_{IC}} \left[\frac{1}{T} - \frac{1}{T_B}\right] dq = V_2 RT_o \int_0^{q_{IC}} \ln[x_A(\alpha - 1) + 1]\, dq, \quad \text{(A2)}$$

where x_A as a function of q is given by

$$y_{A,I} = q x_A + (1 - q)\frac{\alpha x_A}{1 + x_A(\alpha - 1)}. \quad \text{(A3)}$$

For a given feed, relative volatility and the composition of vapor to the condenser ($y_{A,I}$), the thermodynamic efficiency of the distillation column can be calculated if flows through the top section (L_A) and intermediate condenser (V_2) along with parameter q_{IC} are known. In this article, calculations are done for a distillation column that is pinched at both the feed and the intermediate condenser locations (Figure 2b). For a given $y_{A,I}$, the composition of the liquid phase, $x_{A,IC}$, in the two-phase stream exiting the intermediate condenser is calculated from the operating line of the section immediately above the feed:

$$x_{A,IC} = 1 - (1 - y_{A,I})\left[\frac{1 - x_{Af}}{1 - y_{Af}}\right]. \quad \text{(A4)}$$

The composition of the vapor phase in equilibrium with this liquid phase is calculated by the vapor–liquid equilibrium relationship:

$$y_{A,IC} = \frac{\alpha x_{A,IC}}{x_{A,IC}(\alpha - 1) + 1}. \quad \text{(A5)}$$

The vapor flow through the intermediate condenser, V_2, is calculated by adding the vapor contribution from feed to flow rate V_3 in Eq. 7:

$$V_2 = x_{Af}\left[\frac{1 - x_{Af}}{y_{Af} - x_{Af}}\right] + (1 - q_f)(1 - x_{Af}). \quad \text{(A6)}$$

Liquid flow rate in the section immediately above the feed is given by

$$L_2 = V_2\left[\frac{1 - y_{Af}}{1 - x_{Af}}\right]. \quad \text{(A7)}$$

Liquid flow rate, L_A, in the top section of the distillation column is calculated by material balance between the two top sections:

$$L_A = V_2\left[\frac{x_{Af} - y_{Af}}{1 - x_{Af}}\right] \cdot \left[\frac{1 - y_{A,IC}}{x_{A,IC} - y_{A,IC}}\right], \quad \text{(A8)}$$

and parameter q_{IC} is calculated from

$$q_{IC} = \frac{V_2(1 - y_{Af}) - L_A(1 - x_{Af})}{V_2(1 - x_{Af})}. \quad \text{(A9)}$$

Thus thermodynamic efficiency for a pinched column with an intermediate condenser can be easily calculated.

Manuscript received Nov. 17, 1997, and revision received Feb. 27, 1998.